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VELOCITY AND PRESSURE DISTRIBUTIONS IN THE LIQUID RESERVOIR IN A REGENERATIVE LIQUID PROPELLANT GUN

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> > SEPTEMBER 1988



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# 19. ABSTRACT (Con't)

The fundamental equations describing piston motion, injection of the liquid propellant, and the pressure and velocity distributions in the liquid chamber were applied, with simplifications, to an injector model in an earlier paper by the authors. In this paper the authors extend the development to a full treatment of the velocity and pressure distributions. The distributions are developed from continuity and momentum equations which include area change with time and axial position. The Lagrange approximation, i.e. zero density gradient in the liquid, is invoked to simplfy the equations of motion. The resulting PDEs are solved to obtain a time-dependent algebraic equation describing the axial velocity distribution across the liquid reservoir and an integral equation describing the axial pressure distribution. These equations are then solved numerically and the results are compared to solutions from a 1-D model.

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## TABLE OF CONTENTS

		Page
	LIST OF FIGURES	v
I.	INTRODUCTION	1
II.	CONTROL VOLUME	1
III.	BACKGROUND	3
IV.	EQUATIONS OF MOTION FOR THE FLUID AND VELOCITY DISTRIBUTION	6
ν.	PRESSURE DISTRIBUTION	8
VI.	RESULTS	13
VII.	COMPARISON TO ONE-DIMENSIONAL MODEL	18
VIII.	CONCLUSIONS	20
	REFERENCES	21
	LIST OF SYMBOLS	22
	APPENDIX A	23
	DISTRIBUTION LIST	27

## LIST OF FIGURES

<u>Figure</u>		Page
1	-A Regenerative Liquid Propellant Gun with an Annular Piston	2
2	Control Volume	2
3	Comparison of Piston Position vs Time from Simulation (Dotted Line) and Experiment (Solid Line)	4
4	Comparison of Liquid Pressure vs Time from Simulation (Dotted Line) and Experiment (Solid Line)	4
5	Comparison of Discharge Coefficient vs Time from Simulation (Dotted Line) and Experimentally Derived (Solid Line)	5
6	Relative Position of the Piston and the Bolt for the Major Portion of the Injection Cycle	14
7	Predicted Pressure Distribution in the Liquid Reservoir at 3.0 ms	15
8	Predicted Velocity Distribution in the Liquid Reservoir at 3.0 ms	15
9	Relative Position of the Piston and the Bolt for the Beginning Portion of the Injection Cycle	16
10	Predicted Pressure Distribution in the Liquid Reservoir at 1.3 ms	17
11	Predicted Velocity Distribution in the Liquid Reservoir at 1.3 ms	17
12	Comparison of Velocity Distributions at Mid-Stroke from Model (Solid Line) and One-Dimensional Model (Dotted Line)	19
13	Comparison of Pressure Distributions at Mid-Stroke from Model (Solid Line) and One-Dimensional Model (Dotted Line)	19

#### I. INTRODUCTION

The interior ballistic models to date have captured the overall performance of the regenerative liquid propellant gun. Accordingly, recent interest has shifted toward a more complete treatment of the regenerative liquid propellant gun interior ballistic process. In an attempt to describe one of the details of the process, we address the distributions of velocity and pressure in the liquid reservoir of the regenerative liquid propellant gun. The present work is a continuation of the treatment of the liquid injection process presented by the authors 2 in which a simplified treatment of pressure distribution was considered as part of the injection model. The distributions are derived from a modified Lagrange distribution with area change to account for the shape of the regenerative piston and the center bolt. The results are then compared to a one-dimensional simulation.

#### II. CONTROL VOLUME

The regenerative liquid propellant gun process, illustrated in Figure 1, is initiated by firing a primer which pressurizes the initial combustion chamber. The chamber pressure acting on the piston forces it to the rear, compressing the liquid propellant in the reservoir. After an initial transient period, the pressure in the liquid reservoir will be greater than the combustion chamber pressure due to the differential area across the injection piston, resulting in the injection of liquid propellant into the combustion chamber.

In this paper we are concerned only with the liquid propellant reservoir. The combustion chamber pressure is taken from an experimental firing of a 30-mm Concept VI fixture (shown in Figure 1), and input as a boundary condition. The control volume is shown in Figure 2. The contours of the piston and the reservoir are approximated by straight line segments as indicated. The center bolt and the transducer block are fixed in the reference frame of the chamber. The origin of the coordinate system, fixed in the chamber frame of reference, is at the rear (left hand) end of the reservoir, and x is the coordinate along the bolt as shown in Figure 2. The fit ton moves rearward with a velocity  $u_p(t)$ , and the points  $s_1(t)$ ,  $s_2(t)$ , and

 $s_3(t)$  are the coordinates of fixed stations on the inner contour of the piston with respect to the origin, as shown, such that these coordinates vary with time as the piston is displaced to the left. The sign convention for piston velocity is opposite of that for fluid velocity. Note that the right hand face of the control volume coincides with the exit plane of the injection orifice,  $s_3$ , such that the control volume also varies with time.

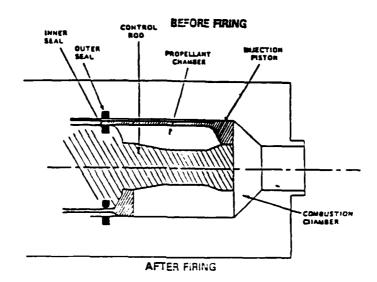
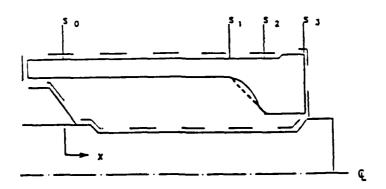


Figure 1. A Regenerative Liquid Propellant Gun with an Annular Piston

## CONTROL VOLUME



CONTROL VOLUME INCLUDES
 RESERVOIR AND PISTON

Figure 2. Control Volume

#### III. BACKGROUND

A simplified treatment of the control volume detailed above which couples the piston motion and the liquid velocity was described in an earlier paper. The goal was to describe the injection process by coupling the motion of the regenerative piston and the liquid injection thereby capturing the time-dependent nature of the discharge coefficient.

A simulation of a firing of a Concept VI 30-mm test fixture at BRL had indicated that the model exhibited a mean value of the discharge coefficient which agreed quite well with that of the experimental data in the steady state operation, but there was lack of agreement in the early values and in the slow rise to steady state. Since the model had not addressed the Belleville springs, which are utilized in the test fixture to permit the piston to clear the seal on the nose of the center bolt, the springs were included in the model with results shown in figures 3, 4 and 5. The comparison between experimental and predicted piston motion, with the inclusion of the Belleville springs, is shown in figure 3. The actual piston travels about 0.55 cm, hesitates as the rear transducer block comes to a stop against the Belleville springs, and then accelerates and smoothly completes its stroke. simulation shows the same qualitative motion as the actual piston. However, the simulated piston position passes through the transient stage more quickly, with the hesitation occurring approximately 1 ms before that in the experiment. The reason for the discrepancy is not clear. In figure 4 a similar result is reflected in the liquid pressure. Qualitatively the simulation reflects the early oscillations in experimental liquid pressure, and, after an initial transient period, remains in close agreement. However, the oscillations occur too early in the simulation. The comparison between the discharge coefficient derived from experimental data and that predicted by the model is shown in figure 5. Although the mean values at steady state are in reasonable agreement, the model does not capture the slow rise to steady state.

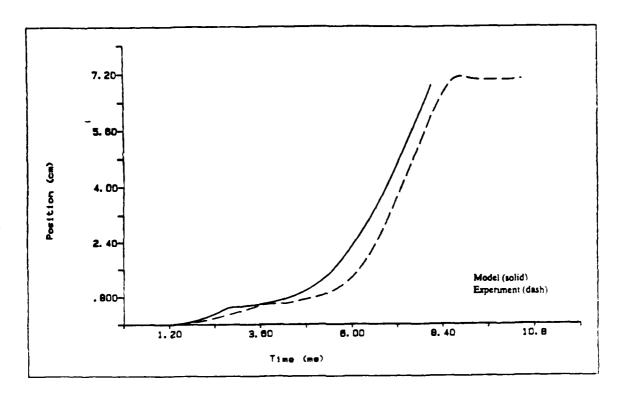


Figure 3. <u>Comparison of Piston Position vs Time from Simulation</u>
(Dotted Line) and Experiment (Solid Line)

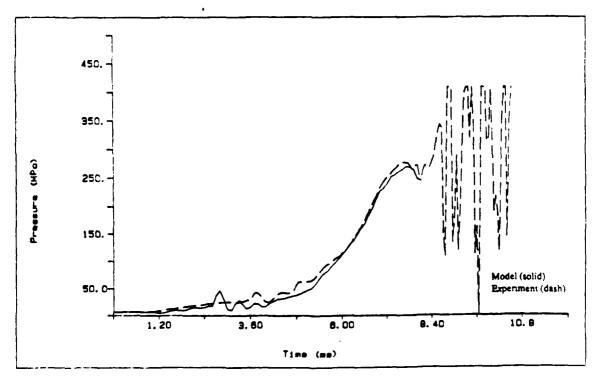


Figure 4. Comparison of Liquid Pressure vs Time from Simulation (Dotted Line) and Experiment (Solid Line)

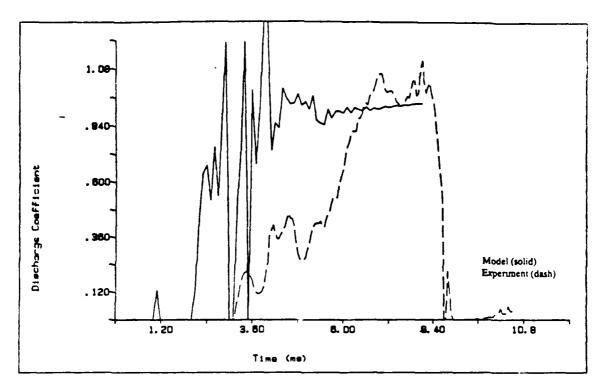


Figure 5. Comparison of Discharge Coefficient vs Time from Simulation (Dotted Line) and Experimentally Derived (Solid Line)

In order to examine the discrepancies between the experiment and the model, several simplifications in the model were examined and removed. The theoretical model can be broken into two major components. The first involves the derivation of equations describing the distributions of velocity and pressure in the liquid reservoir from a continuum analysis. The second involves the coupling of these equations into the momentum equation for the piston. In this paper the first stage of the derivation is discussed, namely the distributions of velocity and pressure in the liquid reservoir. In the earlier model the contours of the outer piston and inner bolt were not included in the representation of the pressure distribution in the liquid reservoir; they are now explicitly considered. It is not possible to evaluate the impact of these additional considerations on the comparison with experimental data until the second stage of the derivation is completed. Therefore, to assess the validity of this stage of the modeling, a comparison is made to a one-dimensional simulation of the liquid reservoir.

#### IV. EQUATIONS OF MOTION FOR THE FLUID AND VELOCITY DISTRIBUTION

The equations of motion for the fluid, continuity and momentum equations, are written to include area change as the piston moves rearward. We note that the area through which the fluid flows is a function of both time and position since the contoured piston moves rearward over a contoured bolt. The equations of motion are then

$$\frac{\partial}{\partial t} (\rho A) + \frac{\partial}{\partial x} (\rho V A) = 0 \tag{1}$$

$$\partial/\partial t (\rho VA) + \partial/\partial x (\rho V^2A) = -A \partial P/\partial x$$
 (2)

We obtain the equation for the velocity distribution from the continuity equation (1). From (1), using the Lagrange approximation, i.e. assuming that the density is spatially uniform at each timestep,  $\partial \rho / \partial x = 0$ , we have

A 
$$(1/p)$$
  $\partial p/\partial t = -\partial A/\partial t - \partial/\partial x$  (VA).

The goal is to eliminate the partial derivatives with respect to t, time, and recast the equation in  $\partial/\partial x$ . The resulting equation is then integrated with respect to x to obtain an expression for velocity as a function of time and position. Consider

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t} \frac{m_L}{V_R} = \frac{\dot{m}_L}{V_R} - \frac{m_L \dot{v}_R}{V_R^2}$$
 (4)

where  $m_{\tilde{L}}$  is the mass of liquid in the reservoir, and  $V_{\tilde{R}}$  is the volume of the reservoir.

Now,

$$\dot{m}_{L} = -\rho A_{3} v_{3} - \rho u_{p} A_{3} \tag{5}$$

and

$$\dot{\mathbf{v}}_{\mathbf{R}} = -\mathbf{u}_{\mathbf{p}} \left( \mathbf{A}_{\mathbf{R}} + \mathbf{A}_{\mathbf{3}} \right) \tag{6}$$

such that, 
$$\frac{1}{\rho} \frac{\partial \rho}{\partial t} = \frac{-[v_3 A_3 - u_p A_R]}{v_R}$$
 (7)

where  $\mathbf{v_3}$  is the velocity of the exiting liquid,  $\mathbf{A_3}$  is the exit vent area,  $\mathbf{A_R}$  is the projected area of the piston on the liquid reservoir side, and  $\mathbf{u_p}$  is the velocity of the piston. The exit vent area,  $\mathbf{A_3}$ , is the annular area between the piston at  $\mathbf{s_3}$  and the bolt.  $\mathbf{A_3}$  is time dependent, but it is not a function of position  $\mathbf{x}$ . The time rate of change of mass includes both the mass flux through the orifice with respect to the bolt and the additional mass flux into the chamber due to the piston motion. It is noted that we have chosen a laboratory reference frame stationary on the bolt with 0.0 at the back wall and the positive x-axis forward. Since the natural motion of the liquid is to the right through the vent, and the natural motion of the piston is to the left toward the back wall,  $\mathbf{v_3}$  is positive to the right while  $\mathbf{u_p}$  is positive to the left.

It can be shown that the time rate of change of area is

$$A(x,t) = u_{p} \partial \Delta x(\pi E^{2}(x,t))$$

where R(x,t) is the radius of the inner contour of the piston where we have explicity noted that area is dependent on position and time.

Equation (3) then becomes

$$\frac{\partial}{\partial x} (vA) = -u_p \frac{\partial \left[\pi R^2(x,t)\right]}{\partial x} + \left[\frac{v_3^A 3}{v_p} - \frac{u_p^A R}{v_p}\right] A(x,t) . \tag{8}$$

Integrating with respect to x and noting that v(0,t) = 0 we obtain

$$v(x,t) = u_{p} \frac{\pi[R^{2} - R^{2}(x,t)]}{A(x,t)} + \frac{[v_{3}A_{3} - u_{p}A_{R}]}{V_{R}} \frac{V(x,t)}{A(x,t)}$$
(9)

where V(x,t) is the volume of the reservoir from the back wall to the position x. Equation 9 defines the velocity distribution in the liquid reservoir.

It is useful to rewrite equation (9) in terms of the liquid areas. Note that

$$\pi[R^{2}_{0} - R^{2}(x,t)] = \pi[(R^{2}_{0} - r^{2}_{0}) - (R^{2}(x,t) - r^{2}_{b}(x))] = A_{W} - A(x,t)$$

where we have used the fact that the radius of the bolt varies by only a small amount over its length, e.g. the maximum radius is 1.79 cm and the mimimum 1.65 cm. Thus  $r_0^2 - r_b^2(x)$  is a very small number. Using this approximation, equation (9) becomes

$$v(x,t) = \frac{u_{p}[A_{W} - A(x,t)]}{A(x,t)} + \frac{[v_{3}A_{3} - u_{p}A_{R}]}{v_{R}} \frac{V(x,t)}{A(x,t)}.$$
 (10)

#### V. PRESSURE DISTRIBUTION

Using equation (1) in (2) and integrating with respect to  $\mathbf{x}$  from 0 to  $\mathbf{x}$  we obtain

$$P(x,t) = P_C(t) - 1/2\rho v^2(x,t) - \rho \int_0^x \dot{v}(x',t) dx'$$
 (11)

in which  $P_{O}(t)$  is the breech pressure at a given time. Integrating the time derivative of (10)

$$\int_{0}^{x} \frac{\partial}{\partial t} v(t,x') dx' = A_{w} \dot{u}_{p} \int_{0}^{x} \frac{1}{A(x,t)} dx' - u_{p}^{2}$$

$$+ u_{p}^{2} A_{w} \frac{1}{A(x,t)} - \dot{u}_{p}^{2} x + z_{2} \int_{0}^{x} \frac{V(x',t)}{A(x',t)} dx'$$

$$- z_{1} u_{p}^{2} A_{w} \int_{0}^{x} \frac{1}{A(x',t)} dx' + z_{1} u_{p}^{2} x - z_{1} u_{p} \int_{0}^{x} \frac{V(x',t)}{A(x',t)} dx'$$
(12)

where

$$z_{1} = \frac{v_{3}^{A_{3}} - u_{F}^{A_{R}}}{v_{R}}$$
 (13)

$$z_{2} = \frac{v_{R}(v_{3}A_{3} + A_{3}v_{3}) - v_{R}A_{R}\dot{u}_{p} + (v_{3}A_{3} - u_{p}A_{R}) u_{p}(A_{R} + A_{3})}{v_{R}^{2}}$$
(14)

and we have used

$$\int_{0}^{x} \frac{1}{A^{2}(x',t)} \frac{\partial A(x',t)}{\partial t'} dx' = \left[\frac{1}{A(x,t)} - \frac{1}{A(x,t)}\right] u_{p}$$
(15)

$$\int_{0}^{\mathbf{x}} \frac{\mathbf{V}(\mathbf{x}',t)}{\lambda^{2}(\mathbf{x}',t)} \frac{\partial \lambda(\mathbf{x}'t)}{\partial t'} d\mathbf{x}' = \left[ \int_{0}^{\mathbf{x}} \frac{\mathbf{V}(\mathbf{x}',t)}{\lambda^{2}(\mathbf{x}',t)} d\mathbf{x}' \right] \mathbf{u}_{p}. \tag{16}$$

By definition

$$\overline{P}(t) = \frac{\int_{C}^{S_3} P(x,t)A(x,t)dx}{\int_{C}^{S_3} P(x,t)A(x,t)dx} = \frac{\int_{C}^{S_3} P(x,t)A(x,t)dx}{V_R}$$
(17)

with  $\bar{P}(t)$  the space-mean pressure.

Thus, the breech pressure  $P_{O}(t)$  is given by

$$P_{O}(t) = \bar{P}(t) + \frac{\rho}{2V_{R}} \int_{0}^{s_{3}} v^{2}(x,t) A(x,t) dx$$

$$+ \frac{\rho}{V_E} \int_{C}^{S_3} A(x,t) \int_{C}^{X} \dot{v}(x',t) dx' dx$$
 (18)

or

$$\begin{split} P_{O}(t) &= \overline{P}(t) + \frac{\rho u}{-2} \frac{2}{V_{R}}^{2} - \int_{0}^{5} \frac{3}{A(x,t)} \int_{0}^{x} \frac{1}{A(x,t)} dx \\ &+ \frac{\rho^{A}}{V_{R}} \frac{u^{2}}{V_{R}} \frac{1}{P} \int_{0}^{5} \frac{3}{A(x,t)} \int_{0}^{x} \frac{1}{A(x,t)} dx^{2} dx - \frac{\rho u}{2} \frac{2}{P} \\ &+ \frac{\rho z}{2V_{R}} - \int_{0}^{5} \frac{3}{A(x,t)} \frac{v^{2}(x,t)}{A(x,t)} dx + \frac{\rho u}{-P} \frac{h^{A}}{V_{R}} \frac{z}{N} \int_{0}^{5} \frac{v(x,t)}{A(x,t)} dx \\ &- \frac{\rho z}{V_{R}} \int_{0}^{5} x A(x,t) dx + \frac{\rho z}{V_{R}} \int_{0}^{5} \frac{3}{A(x,t)} \int_{0}^{x} \frac{v(x,t)}{A(x,t)} dx^{2} dx \\ &- \frac{\rho z}{V_{R}} \frac{z}{N} \int_{0}^{5} 3v(x,t) dx - \frac{\rho z}{V_{R}} \frac{u^{B}}{N} \int_{0}^{5} \frac{3}{A(x,t)} \int_{0}^{x} \frac{v(x,t)}{A(x,t)} dx^{2} dx \\ &- \frac{\rho z}{V_{R}} \int_{0}^{5} \frac{3}{A(x,t)} \int_{0}^{x} \frac{v(x,t)}{A^{2}(x,t)} dx^{2} dx + \frac{\rho z}{V_{R}} \int_{0}^{5} \frac{3}{A(x,t)} \int_{0}^{x} \frac{1}{A(x,t)} dx^{2} dx \\ &- \frac{\rho z}{V_{R}} \int_{0}^{5} \frac{3}{A(x,t)} \int_{0}^{x} \frac{v(x,t)}{A^{2}(x,t)} dx^{2} dx + \frac{\rho z}{V_{R}} \int_{0}^{5} \frac{3}{A(x,t)} dx^{2} dx + \frac{\rho z}{V_{$$

The relationship of breech pressure to P(x,t) is given by

$$P(x,t) = P_{O}(t) - 1/2\rho v^{2}(x,t) - \rho \int_{O}^{x} \dot{v}(x',t) dx'.$$
 (20)

(19)

Combining (10), (12), and (19)

$$P(x,t) = \tilde{P}(t) + \frac{\rho u_p^2 A_w^2}{2V_R} \int_0^{s_3} \frac{1}{A(x,t)} dx$$

$$+ \frac{\rho A_w \dot{v}_p}{V_R} \int_0^S A(x,t) \int_0^X \frac{1}{A(x',t)} dx' dx$$

$$+ \frac{\rho z_1^2}{2V_R} \int_0^{s_3} \frac{V^2(x,t)}{A(x,t)} dx + \frac{\rho u_p^A v_1}{V_R} \int_0^{s_3} \frac{V(x,t)}{A(x,t)} dx$$

$$-\frac{\rho^{2}}{\sqrt{2}}\int_{0}^{s} xA(x,t)dx + \frac{\rho^{2}}{\sqrt{2}}\int_{0}^{s} A(x,t)\int_{0}^{x} \frac{V(x^{\prime},t)}{A(x^{\prime},t)}dx^{\prime}dx$$

$$-\frac{p_{\text{pl}}}{V_{\text{p}}} \int_{C}^{s} V(x,t) dx - \frac{p_{\text{pl}}u_{\text{p}}^{\text{A}}w}{V_{\text{R}}} \int_{C}^{s} A(x,t) \int_{C}^{x} \frac{1}{A(x',t)} dx' dx$$

$$-\frac{cz_1 c}{v_E} \stackrel{s}{\rightarrow} A(x,t) \stackrel{f}{\nearrow} \frac{v(x',t)}{A^2(x',t)} dx'dx$$

$$-\frac{\rho u_{p}^{2} A_{w}^{2}}{2 A^{2}(x,t)} - \frac{\rho V^{2}(x,t) z_{1}^{2}}{2 A^{2}(x,t)} - \frac{\rho u_{p}^{2} A_{w}^{V}(x,t) z_{1}}{A^{2}(x,t)}$$

$$+ \frac{\rho u_{p} V(x,t) z_{1}}{A(x,t)} + [-\rho A_{w} \dot{u}_{p} + \rho z_{1} u_{p} A_{w}] \int_{0}^{x} \frac{1}{A(x',t)} dx'$$

+ 
$$\rho i_F x$$
 -  $\rho z_2 \int_0^x \frac{V(x',t)}{L(x',t)} dx'$  +  $\rho z_1 u_F \int_0^x \frac{V(x,t)}{A^2(x,t)} dx'$ 

$$- \rho z_1 u_p x + - \frac{\rho z_1 u_p}{v_R} \int_0^3 x A(x, t) dx .$$
 (21)

The integrals involve A(x,t), the area at x at a given time, and V(x,t), the volume of the reservoir from the back wall to x at a given time. The integration is performed numerically.

In order to numerically integrate, we first develop expressions for the area, A(x,t), and volume, V(x,t).

Approximating the contour on the inner surface of the piston by the dashed straight line as indicated in Figure 1, we can express the radius of the piston as measured from the center line as

$$R(x,t) = \left\{ R_1 + \frac{R_2 - R_1}{s_2 - s_1} (x - s_1) \left[ 1 - H(s_1 - x) \right] \right\}$$

$$H(S_2 - x) + R_2 [1 - H(S_2 - x)] H(S_3 - x)$$
 (22)

where x indicates the position on the bolt,  $R_1$  indicates the radius of the piston at  $s_1$  and  $R_2$  is the radius of the piston at  $s_2$ . Fix) is the Heaviside function defined as H(x)=0 for x<0 and H(x)=1 for x>=0. The radius of the inner surface of the piston at a position x is time dependent since the piston is moving.

Similarly, the radius of the bolt at any position  ${\bf x}$  on the bolt can be expressed

$$r_b(x) = \{r_1 + \frac{r_2 - r_1}{x_2 - x_1}(x - x_1) [1 - H(x_1 - x)]\} H(x_2 - x) +$$

$$r_2 [1 - H(x_2 - x)] H(x_3 - x)$$
 (23)

The cross-sectional area of the liquid is then given by

$$A(x,t) = \pi R^2(x,t) - \pi r_b^2(x)$$
 (24)

and the volume of liquid, V(x,t), is

$$V(x,t) = \int_{0}^{x} A(x',t) dx'. \qquad (25)$$

Returning to (13), although it is possible to obtain an analytic solution for some of the integrals, the representation is dependent upon the location in the reservoir and the particular geometry of the problem. Thus, to maintain the generality of the solution, it was decided to obtain the values of the integrals numerically. This procedure yields satisfactory results, and it was felt that no further refinement was necessary. The pressure distribution is then given by (21) with the velocity v(x,t) given by (10).

#### VI. RESULTS

The velocity and pressure distributions predicted by the code are reasonable from a fluid dynamics viewpoint, namely, a constriction causes the velocity to rise and the pressure to fall. In particular, the calculated distributions reflect the geometry of the 30-mm test fixture at the Ballistic Research Laboratory.

The predicted velocity and pressure distributions can be classified into one of two cases. For the majority of the injection cycle, the piston head is over the flat portion of the bolt as shown in Figure 6 with points A, B and C denoting fixed locations on the piston as shown. The slant section of the piston is between A and B with the flat vent from B to C. A typical pressure distribution computed by the model is shown in Figure 7 at 3.0 ms where the piston head is positioned over the flat section of the bolt. The pressure distribution follows the shape of the piston as expected; relatively constant along the flat portion of the piston, decreasing over the converging slant section of the piston from A to B, and leveling out again through the vent from B to C. The slight increase in pressure of approximately 1 MPa may indicate difficulty with the numerical integration or differentiation at some timesteps. From the Bernoulli equation, we would expect the velocity

distribution to generally follow the pressure distribution inversely, i.e. as the pressure decreases, the velocity increases. Figure 8 shows the velocity distribution (expressed in scientific notation) at 3.0 ms with the expected result; the velocity gradient is shallow from the back wall of the reservoir to the beginning of the slant section at A, increases from A to B on the converging slant section of the piston, and maintains a relatively constant velocity through the vent. Although the velocity appears to be zero over the flat section of the piston, a more detailed look at the velocity distribution shows a linear increase from zero at the back wall to approximately 60 cm/s at position A, the beginning of the slant section of the piston.

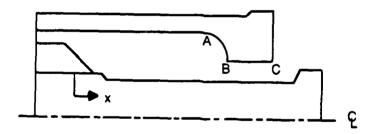


Figure 6. Relative Position of the Piston and the Bolt for the Major
Portion of the Injection Cycle

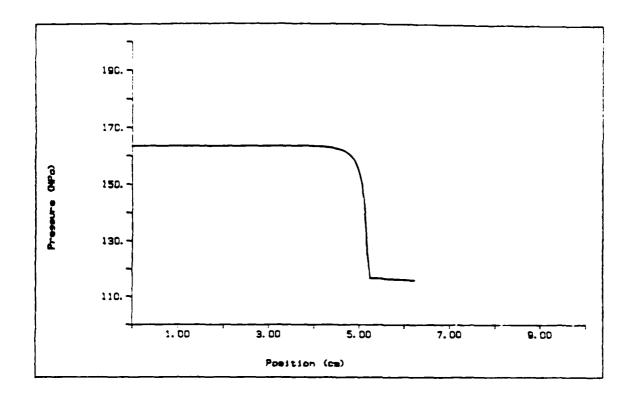


Figure 7. Predicted Pressure Distribution in the Liquid Reservoir at 3.0 ms

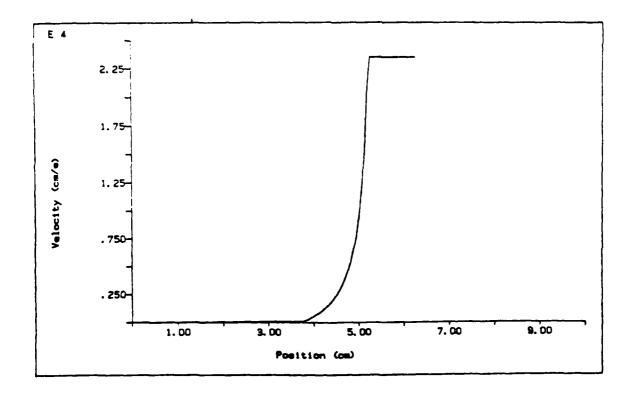


Figure 8. Predicted Velocity Distribution in the Liquid Reservoir at 3.0 ms

A second case of piston position relative to the bolt is shown in Figure 9. During the startup cycle the piston is positioned such that the vent from B to C passes over the slant section of the bolt from D to E. Thus, exiting liquid encounters a converging area from A to B, a flat section of the vent, and an additional converging area when C is between D and E. It is expected that the predicted pressure and velocity distributions will reflect the additional area change. The pressure distribution at 1.3 ms is shown in Figure 10. Again the pressure is stable over the flat section of the piston, drops between points A and B, and flattens through the vent. However, as the liquid encounters the convergent area on the bolt from D to E, the pressure distribution shows an additional drop. The velocity distribution in the reservoir at 1.3 ms, Figure 11, again shows the expected relationship between the velocity and pressure distributions.

The magnitude of the velocity and pressure gradients at 1.3 ms is lower than at 3.0 ms, reflecting the lower piston velocity near the beginning of the injection cycle.

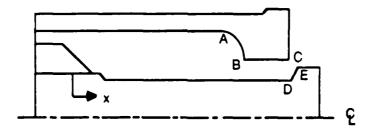


Figure 9. Relative Position of the Piston and the Bolt for the Beginning
Portion of the Injection Cycle

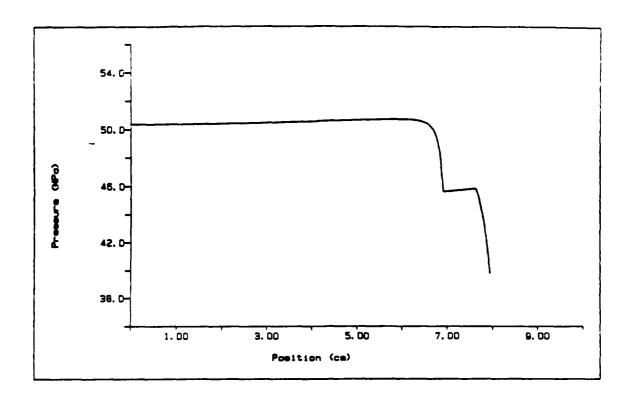


Figure 10. Predicted Pressure Distribution in the Liquid Reservoir at 1.3 ms

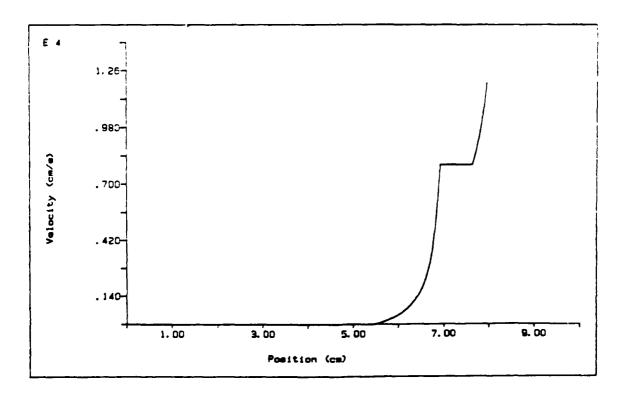


Figure 11. Predicted Velocity Distribution in the Liquid Reservoir at 1.3 ms

#### VII. COMPARISON TO ONE-DIMENSIONAL MODEL

A comparison of the predicted velocity and pressure distributions to a one-dimensional model developed by Coffee<sup>4</sup> gives a quantitative assessment of the model. The one-dimensional representation differs from the model described in this paper in that the back wall is moved to simulate piston motion as taken from an experimental firing of the 30-mm test fixture at the BRL. Since the velocity of the back wall is non-zero in the one-dimensional model, the piston velocity was subtracted from the 1-D velocity distribution for purposes of direct comparison. The piston positions were matched with respect to the fixed coordinate system on the bolt at a time of 3.4 ms in both models.

The comparison of the velocity distributions is shown in Figure 12. Quantitatively the two models show good agreement on the flat section of the piston and the vent with a difference in exit velocities of about 1%. The major discrepancy between the models occurs on the slant section of the piston. This is felt to reflect the approximation of the partial differential equations with ordinary differential equations.

The comparison of pressure distributions between the model and one-dimensional simulation is shown in Figure 13. The gradients show good qualitative and quantitative behavior with a difference of .2% at the back wall.

Future work will focus on integration of these equations into the complete injector model.

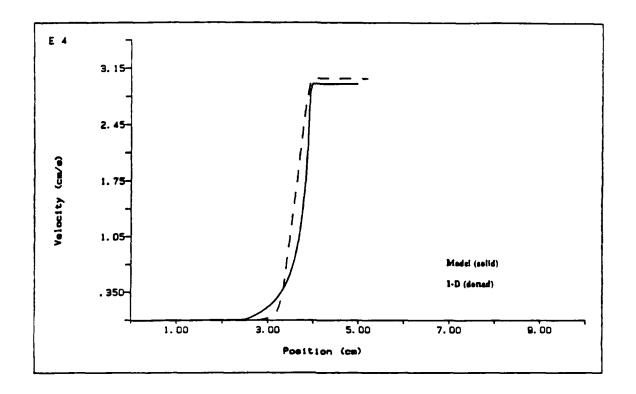


Figure 12. Comparison of Velocity Distributions at Mid-Stroke from Model (Solid Line) and One-Dimensional Model (Dotted Line)

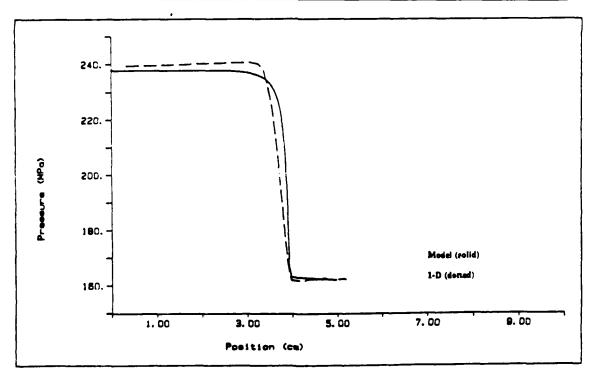


Figure 13. <u>Comparison of Pressure Distributions at Mid-Stroke from Model (Solid Line) and One-Dimensional Model (Dotted Line)</u>

#### VIII. CONCLUSIONS

A simplification of the continuity and momentum equations yields equations to describe the velocity gradient and pressure gradient in the liquid reservoir of a regenerative liquid propellant gun. The resulting equations are applicable to other time-dependent geometries provided the assumption that  $\partial \rho/\partial x=0$  is acceptable. When applied to the 30-mm test fixture at the BRL, the velocity and pressure distributions are reasonable from a fluid dynamics perspective and compare favorably with results from a one-dimensional code. Future work will concentrate on fully exploiting the pressure distribution equation in modeling the relationship between piston movement and liquid flow from the reservoir.

#### REFERENCES

- 1. Morrison, W.F. and Wren, G.P., "A Lumped Parameter Description of Liquid Injection in a Regenerative Liquid Propellant Gun," Proceedings of the 23rd JANNAF Combustion Meeting, October 1986.
- Morrison, W.F. and Wren, G.P., "A Model of Liquid Injection in a Regenerative Liquid Propellant Gun," Ballistic Research Laboratory Technical Report, BRL-TR-2851, July 1987.
- 3. Coffee, T., "Numerical Modeling of Injection in a Liquid Regenerative Propellant Gun," Ballistic Research Laboratory Technical Report, BRL-TR-2897, March 1988.

#### LIST OF SYMBOLS

Cross-sectional area of the vent, cm<sup>2</sup> Cross-sectional area of the liquid,  $cm^2$ Cross-sectional are of the piston on reservoir side,  ${\rm cm}^2$ Cross-sectional area of the piston at x,  $cm^2$ 'A(x,t) Cross-sectional area of the piston at the back wall,  ${\rm cm}^2$ Mass flux of liquid into the combustion chamber, gm/s Combustion chamber pressure, MPa P3 Ē Space mean pressure, MPa Radius of the bolt at the back wall, cm  $\mathbf{r}_{0}$ Radius of the bolt at x, cm  $r_{b}(x)$ R(x,t) Radius of the pistor at x, cm Velocity of the piston, cm/s  $u_p$ Acceleration of the piston, cm/s<sup>2</sup> Velocity of the liquid at s3, cm/s **v**<sub>3</sub> Acceleration of the liquid at  $s_3$ ,  $cm/s^2$ ٧̈́3 Volume of the liquid reservoir, cm<sup>3</sup> Density of the liquid at a given time, gm/cm

## APPENDIX A

The following data base was utilized to obtain the reported results.

#### Concept 6, 30-mm fixture -- Rd 8 after springs

```
COMBUSTION CHAMBER AREA -
                             44.84700
PISTON AREA--C CH SIDE -
                             34.32600
PISTON AREA--RES SIDE -
                             23.27800
                             1.43200
LENGTH L PRIME
LENGTH OF VENT
                             1.04000
PISTON OFFSET
                             0.54400
                        - 2109.20000
PISTON MASS
VOLUME LIQUID
                          172.62764
VENT OPTION
STRAIGHT LENGTH OF PIST -
                              5.94680
MAX PISTON TRAVEL
                              7.37880
DENSITY LIQUID
                              1.43700
K1
                          5350.00000
K2
                              9.11000
FRICTION LOSS LIQ OPTION-
FRICTION LOSS PIS OPTION-
                              0
TIME-C CH PRES DATA FILE: ptoff64.dat
GEOMETRY DATA FILE:
                          r8geo55.dat
GRAPH DATA FILE:
                          30mmc6.gra
INITIAL PR IN RESERVOIR -
                             29.00000
INITIAL VEL IN VENT
                            0.00000
INITIAL PISTON VELOCITY -
                           358.00000
INITIAL PISTON POSITION =
                           0.00000
INTEGRATOR - - TINC
                             0.00010
INTEGRATOR - - EPS
                            0.00001
INTEGRATOR - - METH
                             1
INTEGRATOR - - MITER
                              0
INTEGRATOR - - KWRITE
DIFFERENTIAL EQUATION SET: 1
RAD PIST3 -
             1.83000
                          RAD PIST2 =
                                       1.83000
                                                   RAD PIST1 =
                                                                  3.28000
RAD BOLT3 -
            1.79770
                         RAD BOLT2 -
                                       1.79770
                                                   RAD BOLT1 -
                                                                  1.65000
VOL FUEL12 - 17.90837
```

VOL FUEL23 = 2.02661

FLAT LEN BOLT - 0.55880 SLANT LEN BOLT - 0.76000 IWRITE - 0

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